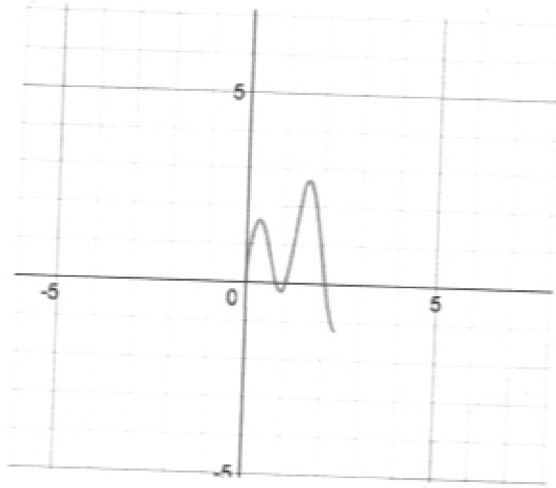
Deriving 2 Fast and 2 Flawlessly

1.) What is The function?

$$\cos x^1 - \cos x^2 + \sin x^{\frac{1}{4}} + \sin 5x \quad \{x < 2.364\}$$



2. What are the Minimums and Maximums?

① Identify The first derivative

$$\cos x + \cos(x^2) + \sin(x^{\frac{1}{4}}) + \sin(5x)$$

$$-\sin x \quad -\sin x^2 \cdot 2x \quad \cos x^{\frac{1}{4}} \cdot \frac{1}{4x}^{\frac{-3}{4}} \quad \cos 5x \cdot 5$$

$$-\sin x + \sin x^2(2x) + \frac{1}{4x}^{\frac{-3}{4}} \cos x^{\frac{1}{4}} + 5x\cos 5x$$

② Plug into Desmos - Identify where $f(x)=0$ (when points intersect The x-axis)

$$\left.\begin{aligned} x &= .-34 \\ x &= .866 \\ x &= 1.655 \\ x &= 2.313 \end{aligned}\right\} \text{minimum/maximums for equation.}$$

③ To identify which is The minimum and maximum identify 2nd derivative.

incorrect rules used

$$-\sin x + \sin x^2(2x) + \frac{1}{4x}^{-3/4} \cos x^{\frac{1}{4}} + 5x\cos 5x$$

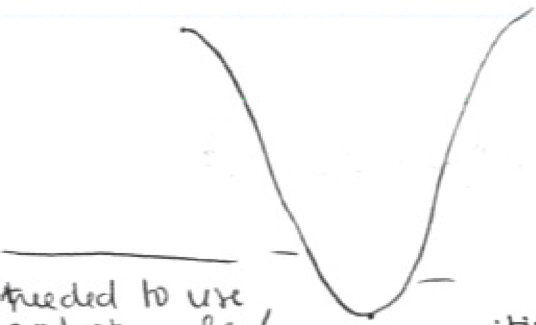
wrong... ☹

$$-\cos x \quad + 2(2x^2\cos(x^2) + \sin x^2) + -x^{\frac{1}{4}}\frac{\sin(x^{\frac{1}{4}}) - 3\cos(x^{\frac{1}{4}})}{16x^{\frac{7}{4}}}$$

$$-25\sin(5x)$$

Then plug values into 2nd derivatives

MINIMUMS | MAXIMUMS

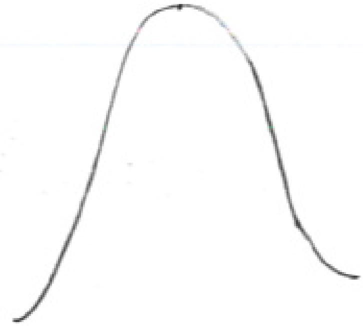


.866

2.313

needed to use product rule / chain rule for 2nd derivative !

positives



negatives

-.34

-1.6555

didnt properly plug in numbers because of incorrect 2nd derivative

3 What is the precise speed at an exact moment on your function?

$$f(x) = \cos x - \cos x^2 + \sin x^{\frac{1}{4}} + \sin 5x \to f'(x) = -\sin x + 2x\sin(x^2)\,\tfrac{1}{4}x^{-3/4} \cos(x^{\frac{1}{4}}) + 5\cos(5x)$$

$$-\sin(2) + 2(2)\sin(2^2) + \tfrac{1}{4}x^{-3/4}\cos(x^{\frac{1}{4}}) + 5\cos(5(2))$$

$$-\sin 2 + 4\sin 4 + \tfrac{1}{4}(2)^{-3/4}\cos(2^{\frac{1}{4}}) + 5\cos 10$$

$$-\sin 2 + 4\sin 4 + \frac{1}{2}^{-3/4}\cos 2^{\frac{1}{4}} + 5\cos 10$$

4 Provide one more mathematical detail of your choice about your function.

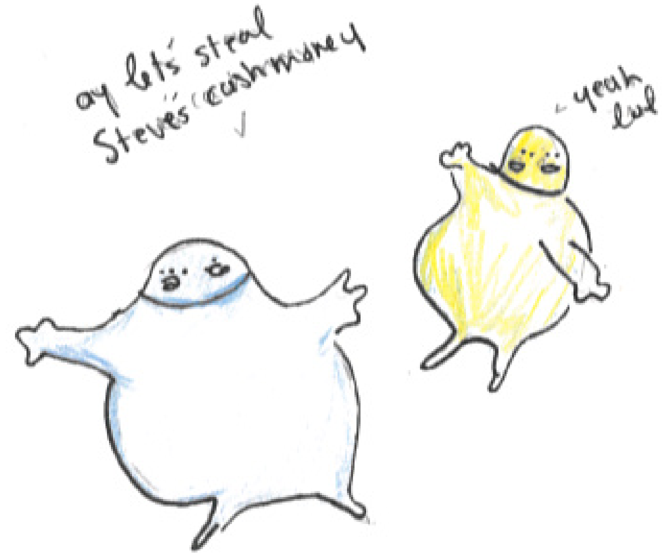
$$f'''(x) = \sin x + 2(6x\cos(x^2) - 4x^3\sin(x^2)) + \frac{9x\sin(x^{\frac{1}{4}}) + 21x^{\frac{3}{4}}\cos(x^{\frac{1}{4}}) - x^{\frac{5}{4}}\cos x^{\frac{1}{4}} - x^{\frac{5}{4}}\cos(x^{\frac{1}{4}})}{64x^{\frac{7}{2}}} - 125\cos(5x)$$

Should have used 2nd derivative

Chenoa Levy

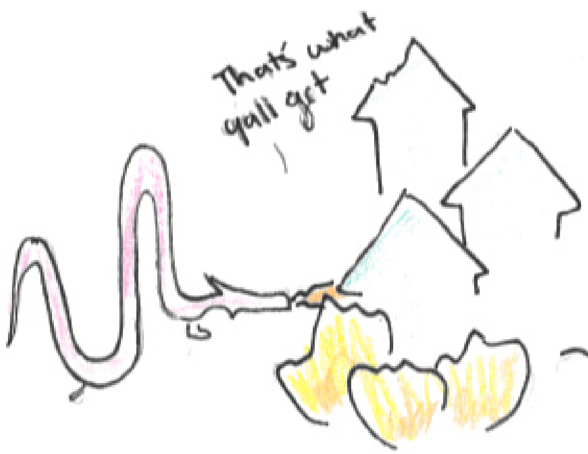
Chenva Levy

There was a very wealthy dragon named Steve who lived in a cave. He spent his day pillaging local towns. He began building his collection of gold and possessions



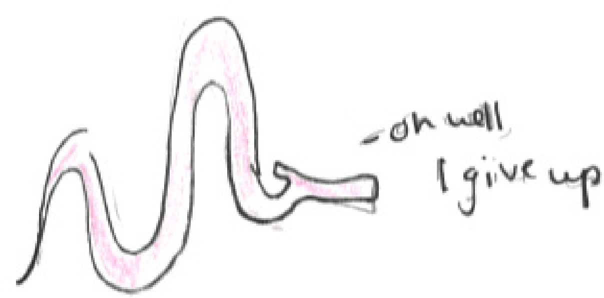
The Boobah clan became jealous of Steve's weath and plotted to steal it from him.

When Steve came back from his day at the spa he realized that all his gold was gone and his cave was wrecked so now he was in debt.



Outraged Steve decided to attack the village and reclaimed his gold and take a little extra something for his troubles.

The villagers were tired of Steve's antics and banned together to raid Steve's cave.



Once again All of Steve's possessions were stolen But this time Steve decided to give up and stop stealing.

Clarissa Levy

$-\sin + \underbrace{\sin x^2}_{u}\underbrace{(2x)}_{v} + \underbrace{\tfrac{1}{4}x^{-3/4}}_{u}\underbrace{\cos x^{\frac14}}_{v} + \underbrace{5x}_{u}\cdot\underbrace{\cos 5x}_{v}$

$-\cos + uv' = \sin x^2(2)$

$vu' \quad 2x(2x\cos(x^2)$

$u' = 2x\cos(x^2)$

$v' = 2$

$\underline{-\cos} + \underline{= 2\sin x^2 + 4x^2\cos x^2}$

$u = \tfrac14 x^{-3/4}$ $\quad -\cos x^{\frac14} \cdot x^{\frac14}$

$v \quad \cos x^{\frac14}$

$u' = -3/4\, x^{-7/4}$

$v' = \cancel{\tfrac14 \sin x^{-3/4}}$ $\tfrac14 x^{-3/4} \sin x^{\frac14}$

$uv = \tfrac14 x^{-3/4}(-3/4 \sin x^{\frac14})$

$vu = \cos x^{\frac14}$ ~~…~~ $(-3/4x)$

$\underbrace{5x\cos}_{u}\underbrace{5x}_{v}$

$u = 5x\cos$

$v = 5x$

$u_1 = 5x \cdot \cos = -\sin 5 \quad 5x - \sin$

$v' = 5$

$uv = 5x\cos(5) = 25x\cos$

$uv = 5x(-\sin 5x) = -\sin 25x^2$

$\cancel{25x\cos - \sin 25x}$

$25x\cos - \sin 25x^2$

$-\cos + 2\sin x^2 - 4x^2\cos x^2 \quad + 25x\cos\sin 25x^2$

MIKAYLA
ELLIOT

Story Function

$\boxed{\text{FUNCTION HAS AN ASYMPTOTE AT 0.}}$

Equation:

$$f(x) = 4x^{-2} - 2x$$

$$\downarrow$$

$$f'(x) = -8x^{-3} - 2$$

$$\downarrow$$

$$f''(x) = 24x^{-4}$$

Extrema:

$$f'(x) = -8x^{-3} - 2 = 0$$

$$\downarrow$$

$$f'(x) = -8x^{-3} = 2$$

$$\downarrow$$

$$f'(x) = x = -2\tfrac{2}{3}$$

Maximum or Minimum:

$$f''(x) = 24x^{-4}$$

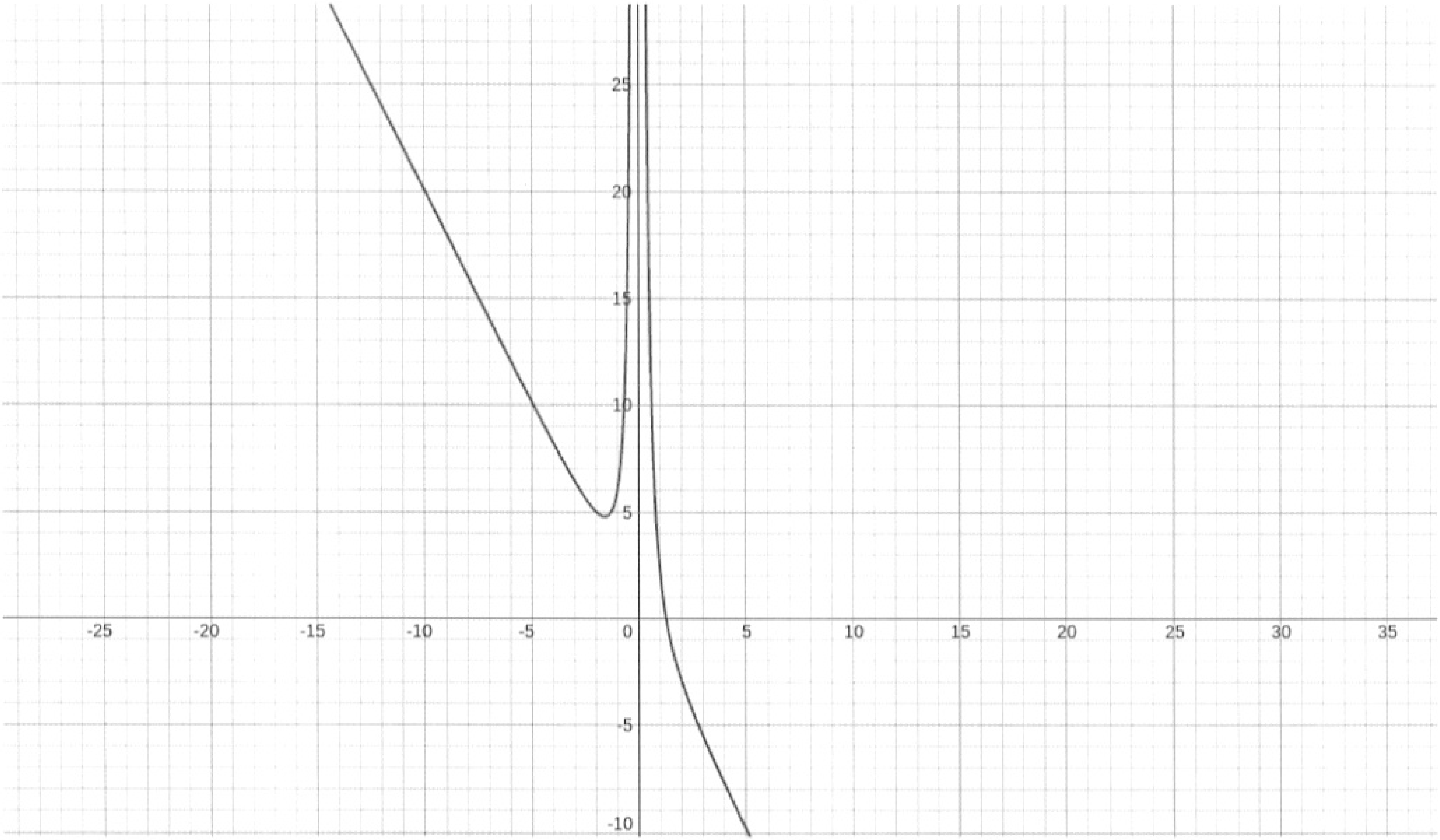
$$\downarrow$$

$$f''(x) = 24(-2\tfrac{2}{3})^{-4} \longrightarrow 24(0.189)$$

$$\downarrow$$

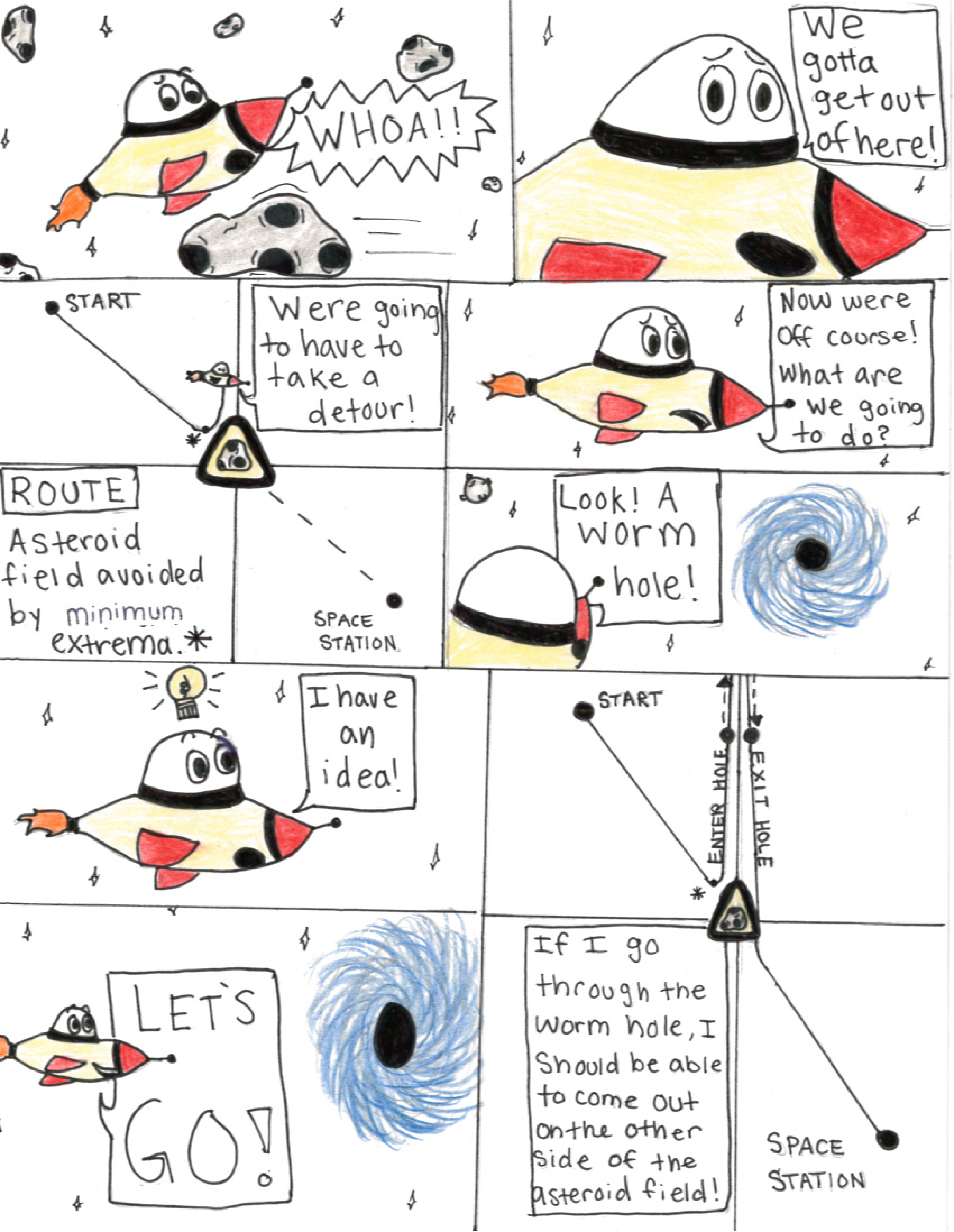
$$\text{Minimum} \longleftarrow 4.536$$

 MIKAYLA ELLIOT



$f(x) = 4x^{-2} - 2x$

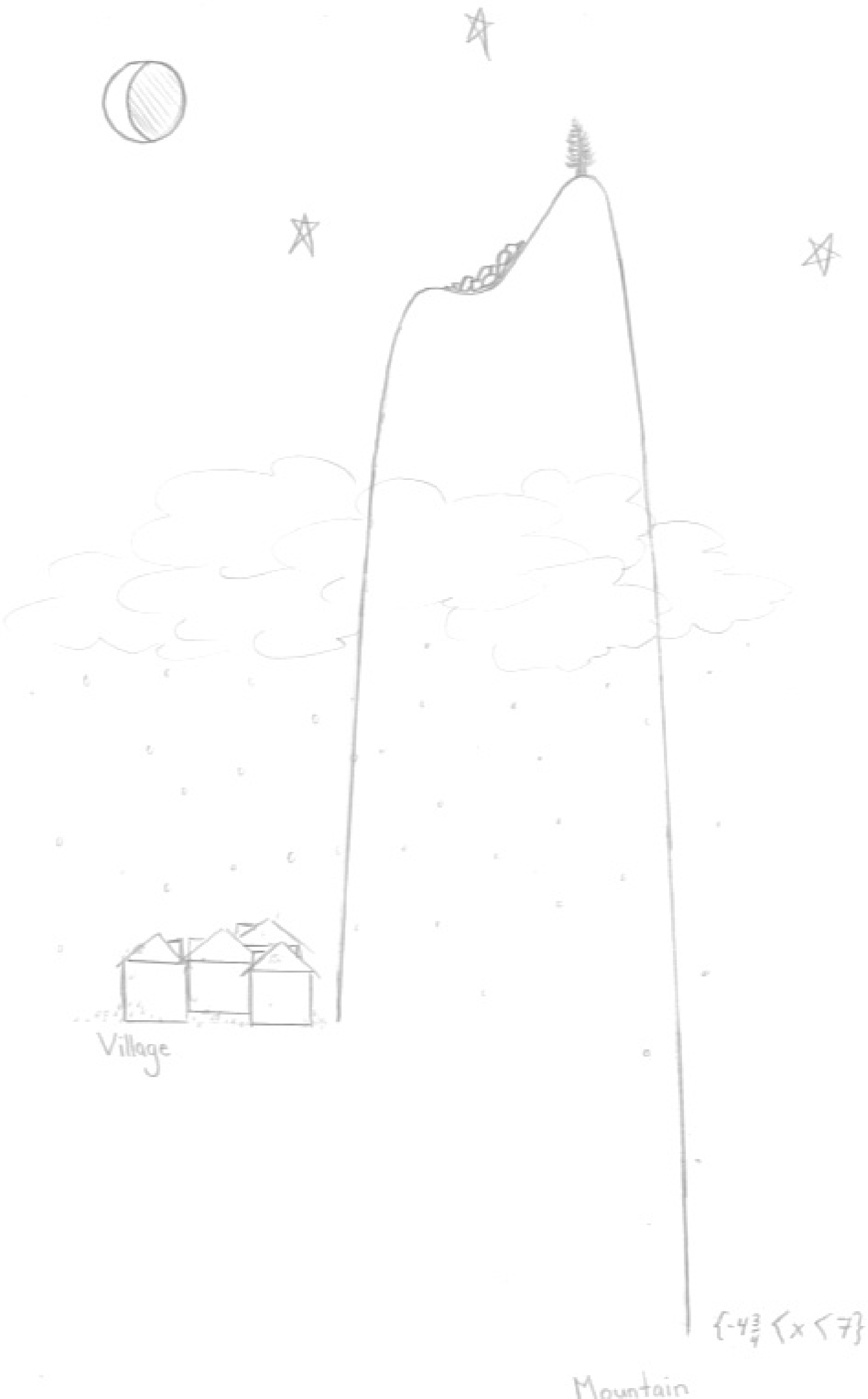

ALFREDS BIG ADVENTURE
BY: MIKAYLA ELLIOT
Hi! I am Alfred! Today we will be flying to the space station across the quadrant!
START
This is the current route that I am taking!
SPACE STATION
The trip should take about 4 years in total!
LET'S GO!!
2 YEARS....
We are about half way there!
Oh, no! Asteroids are coming very fast!


WHOA!!
We gotta get out of here!
START
Were going to have to take a detour!
Now were off course! What are we going to do?
ROUTE
Asteroid field avoided by minimum extrema.*
SPACE STATION
Look! A worm hole!
I have an idea!
START
ENTER HOLE
EXIT HOLE
LET'S GO!
If I go through the worm hole, I should be able to come out on the other side of the asteroid field!
SPACE STATION


LOOK! It's the other end of the worm hole!
WOW!!!
We made it!
There is the Station!
Thank You for all of your help! we wouldn't have made it without you!
UNTIL NEXT TIME...

This is Barry.

$$f(x) = \left[-\frac{1}{25}(x^2)(x+2)(x-5)\right] + 11$$



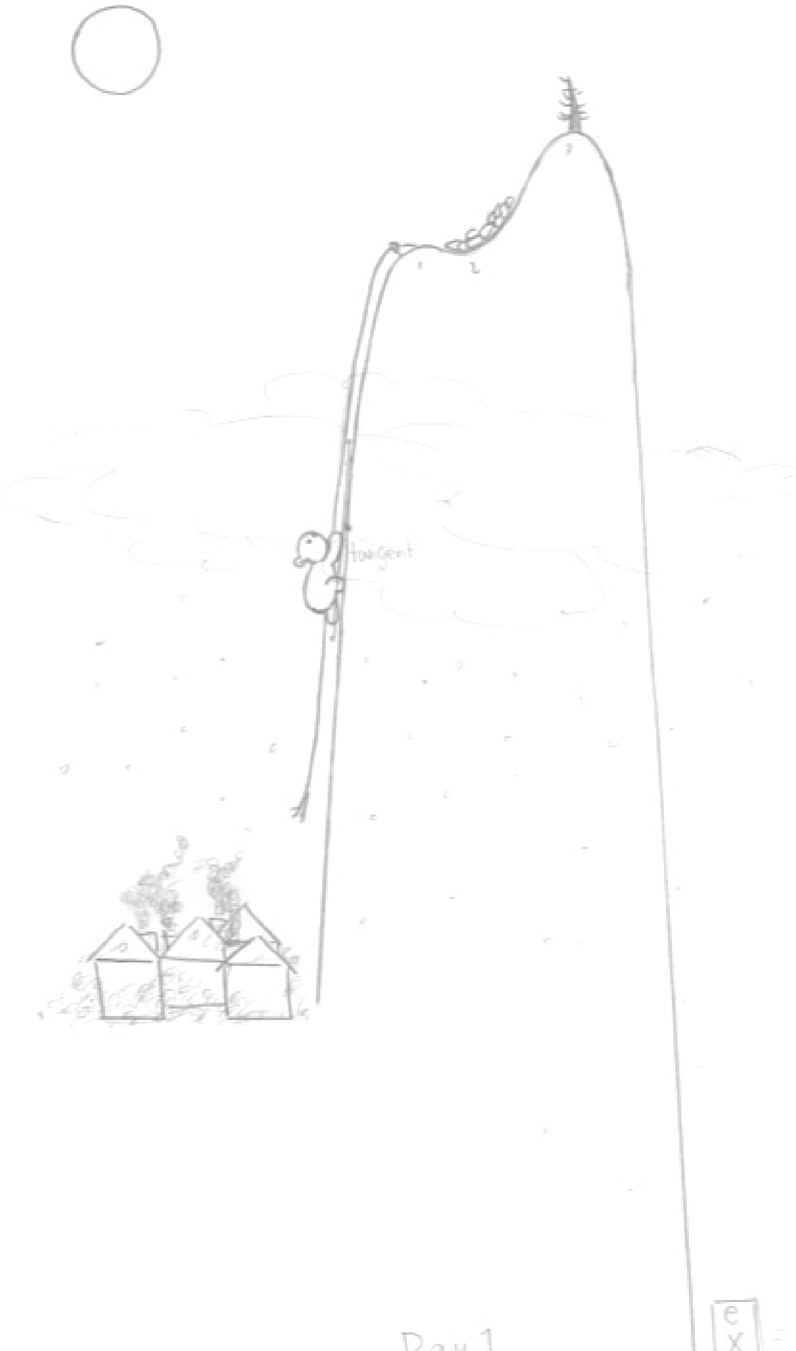
Barry lived in a village at the base of a tall mountain. It always snowed in Barry's village, and he never saw the sun, moon or stars.

$$f'(x) = \frac{-4}{25}x^3 + \frac{9}{25}x^2 + \frac{4}{5}x^1$$

$$\left(-\frac{4}{25}\cdot 3\right)x^{(3-1)} + \left(\frac{9}{25}\cdot 2\right)x^{(2-1)} + \left(\frac{4}{5}\cdot 1\right)x^{(1-1)}$$

→ 2nd derivative. Power rule

→ 1st derivative



Day 1

extremas

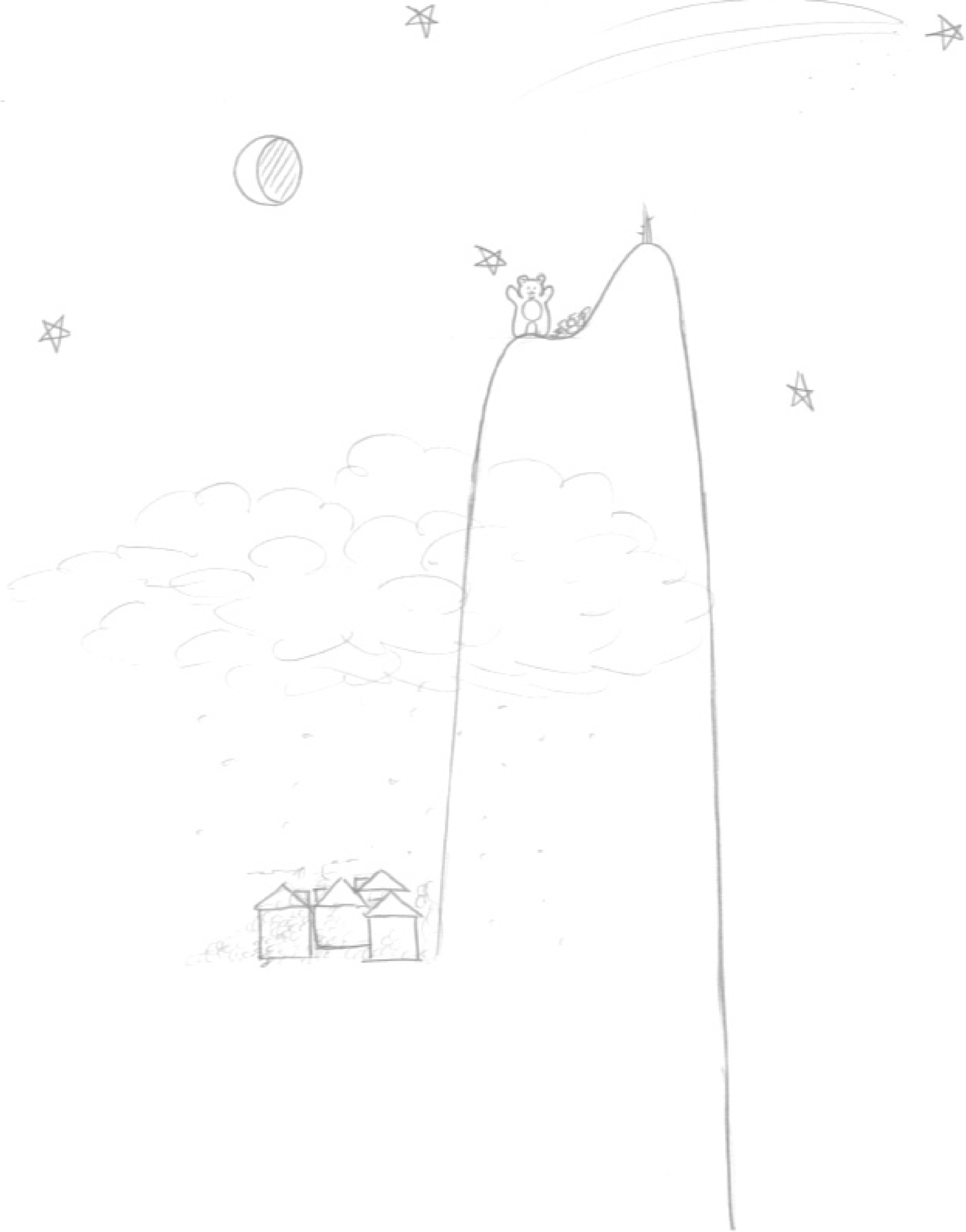
relative max. -1.378123049

absolute max. 2.628123049

min. Ø

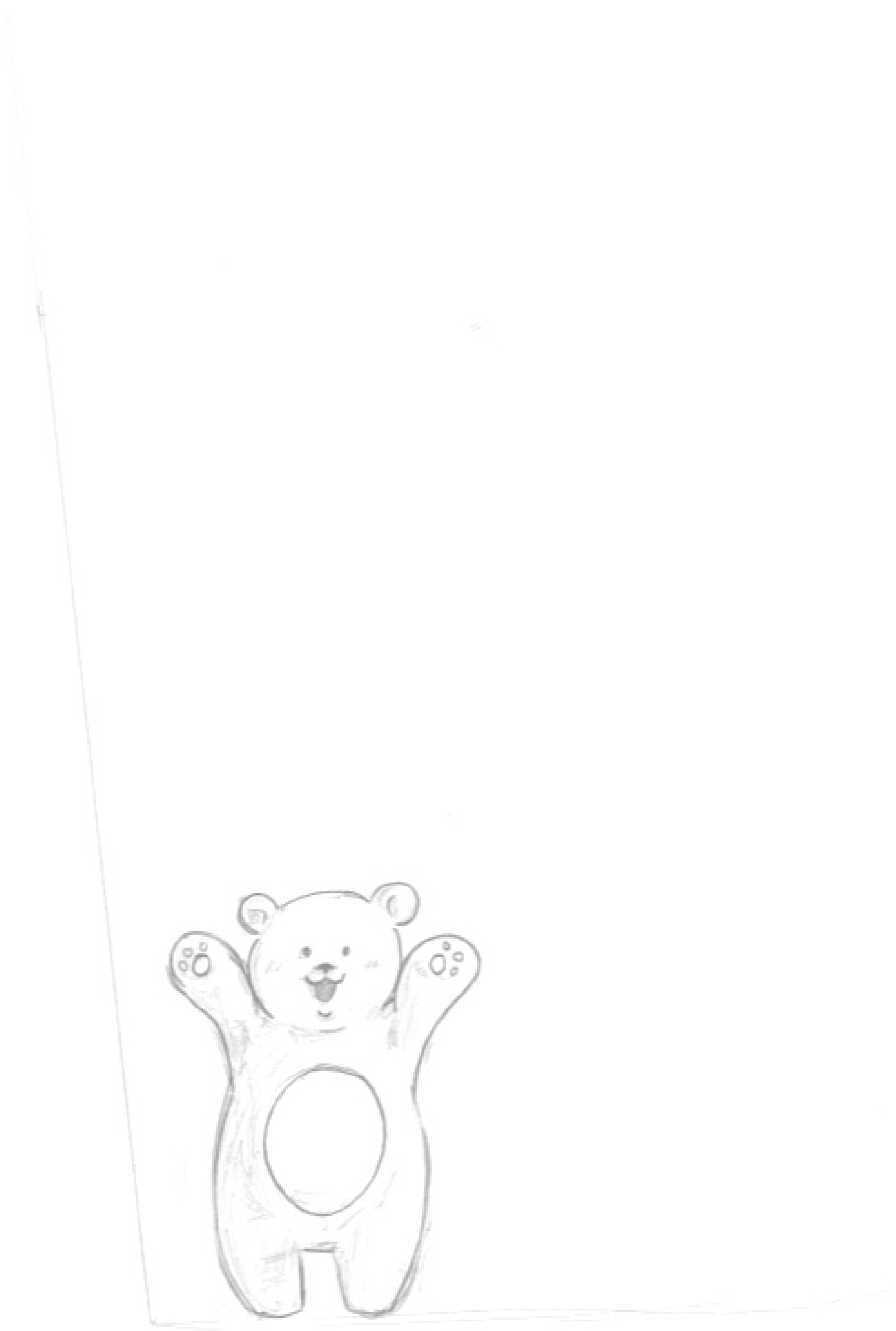
No one had ever climbed the mountain next to Barry's village, so Barry decided to climb the mountain.

$$f''(x) = -\frac{12}{25}x^2 + \frac{18}{25}x + \frac{4}{5}$$



A little more than three days later, Barry reached the first peak of the mountain. Just before the eighth day of Barry's journey he reached the second peak. He finally saw the moon and stars

The next day, Barry saw the sun. Noticing how far he had already gone, Barry decided to continue down the mountain



It took Barry $11\frac{3}{4}$ days to reach the base of the other side of the mountain.

Diana Rothman
1 May 2016

Deriving 2 Fast and 2 Flawlessly

Equation

$f(x) = [-1/25 (x^2)(x+2)(x-5)] + 11$

$\hookrightarrow [-1/25 (x^3 - 2x^2)(x-5)] + 11$

$\hookrightarrow [-1/25 (x^4 - 5x^3 + 2x^3 - 10x^2] + 11$

$\hookrightarrow [-1/25 (x^4 - 3x^3 - 10x^2)] + 11$

$\hookrightarrow f(x) = -1/25\, x^4 + 3/25\, x^3 + 2/5\, x^2 + 11$ ✓

expl. This work above shows my original equation/function for my story. Below, I will use the Power Rule to find the first and second derivatives. In this case, I will start by multiplying $-1/25$ by 4, subtracting 1 from 4, and so on.

1st d.

$f(x) = -1/25\, x^4 + 3/25\, x^3 + 2/5\, x^2 + 11$

$\hookrightarrow (-1/25 \cdot 4)x^{(4-1)} + (3/25 \cdot 3)x^{(3-1)} + (2/5 \cdot 2)x^{(2-1)} + (11 \cdot 0)x^0$

$\hookrightarrow f'(x) = -4/25\, x^3 + 9/25\, x^2 + 4/5\, x$ ✓

2nd d.

$f'(x) = -4/25\, x^3 + 9/25\, x^2 + 4/5\, x$

$\hookrightarrow (-4/25 \cdot 3)x^{(3-1)} + (9/25 \cdot 2)x^{(2-1)} + (4/5 \cdot 1)x^{(1-1)}$

$\hookrightarrow f''(x) = -12/25\, x^2 + 18/25\, x + 4/5$ ✓

expl. The first derivative will help me find the extremas of my function, as well as the slope of a point or points at a given time in the function.

Restrictions

$(-4.75 < x < 7)$

expl. The above restrictions mean that my graph starts at the x-value -4.75 and ends at the x-value 7.

1st d. + extremas

$f'(x) = -4/25\, x^3 + 9/25\, x^2 + 4/5\, x$

$\hookrightarrow -4/25\, x^3 + 9/25\, x^2 + 4/5\, x = 0$

$\hookrightarrow (x)(-4/25\, x^2 + 9/25\, x + 4/5) = 0$

$\hookrightarrow (0)(-4/25(0)^2 + 9/25(0) + 4/5) = 0$ ✓

EXTREMAS

$\boxed{x = 0,\ -1.378123049,\ 3.628123049}$

$$\longrightarrow \frac{-b \pm \sqrt{(b)^2-4ac}}{2a}, \quad (x)\left(-\tfrac{4}{25}x^2+\tfrac{9}{25}x+\tfrac{4}{5}\right)$$

$$\boxed{\begin{aligned} a &= -4/25 \\ b &= 9/25 \\ c &= 4/5 \end{aligned}}$$

$$\hookrightarrow \frac{-(9/25) \pm \sqrt{(9/25)^2 - 4(-4/25)(4/5)}}{2(-4/25)}$$

$$\hookrightarrow \frac{-9/25 \pm \sqrt{81/625 - 4(-16/125)}}{-8/25}$$

$$\hookrightarrow \frac{-9/25 \pm \sqrt{81/625 + 64/125}}{-8/25}$$

$$\hookrightarrow \frac{-9/25 \pm \sqrt{401/625}}{-8/25}$$

$$(+) \quad \frac{-9/25 + 0.8009993758}{-8/25} \longrightarrow \frac{0.4409993758}{-8/25} \longrightarrow -1.378123049 \checkmark$$

$$(-) \quad \frac{-9/25 - 0.8009993758}{-8/25} \longrightarrow \frac{-1.160999376}{-8/25} \longrightarrow 3.628123049 \checkmark$$

exp. By setting the first derivative equal to 0 and solving for x, it is possible to find the values of the extremas. In this case, the values are 0, -1.378123049, and 3.628123049. After talking to my brother, he had me plug the extrema values back into the original equation to find the relative and absolute extremas.

← this is not the (x,y) pt. of the extrema...

- $-1/25(-1.378123049)^4 + 3/25(-1.378123049)^3 + 2/5(-1.378123049)^2 + 11$
  $\hookrightarrow -1/25(3.607048529) + 3/25(-2.617363182) + 2/5(1.899223138) + 11$
  $\hookrightarrow -.1442819412 - .3140835818 + .7596892552 + 11 = 11.30132373 \checkmark$

- $-1/25(0)^4 + 3/25(0)^3 + 2/5(0)^2 + 11 = 11 \checkmark$

- $-1/25(3.628123049)^4 + 3/25(3.628123049)^3 + 2/5(3.628123049)^2 + 11$
  $\hookrightarrow -1/25(173.2718577) + 3/25(47.75798817) + 2/5(13.16327686) + 11$
  $\hookrightarrow -6.930874306 + 5.730958581 + 5.265310743 + 11 = 15.06539502 \checkmark$

exp. Because 11 is the lowest value, this is my absolute minimum extrema.

Because 15.06... is my greatest value, this is the absolute maximum extrema. Thus, the extrema valued at -1.378123049 is only a relative maximum extrema.

2nd d. test $f''(x) = -\frac{12}{25}x^2 + \frac{18}{25}x + \frac{4}{5}$

- $-\frac{12}{25}(-1.378123049)^2 + \frac{18}{25}(-1.378123049) + \frac{4}{5}$
  $\hookrightarrow -\frac{12}{25}(1.899223138) - .9922485953 + \frac{4}{5}$
  $\hookrightarrow -.9116271062 - .9922485953 + \frac{4}{5} = -1.103875702 < 0$ ✓

expl. Here I plugged in the first extrema value I found into the second derivative. The resulting value, which is less than 0, tells me that this extrema is a localized maximum value because the concavity of the graph opens downwards (↶). When the value of a number entered into the second derivative is greater than 0, then the concavity of the graph opens upwards and the extrema is a minimum.

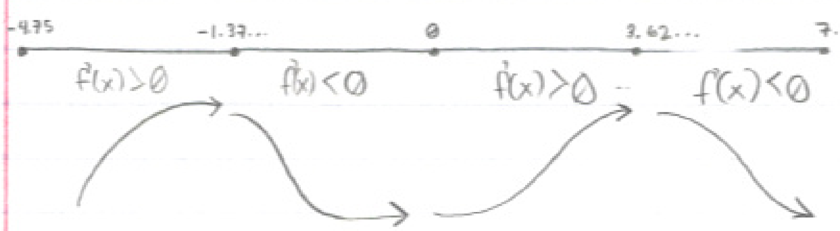
- $-\frac{12}{25}(0)^2 + \frac{18}{25}(0) + \frac{4}{5} = \frac{4}{5} > 0$ ✓

expl. This is a localized minimum extrema opening upwards.

- $-\frac{12}{25}(3.628123049)^2 + \frac{18}{25}(3.628123049) + \frac{4}{5}$
  $\hookrightarrow -\frac{12}{25}(13.16327686) + 2.612248595 + \frac{4}{5}$
  $\hookrightarrow -6.318372892 + 2.612248595 + \frac{4}{5} = -2.906124297 < 0$ ✓

expl. This part of the graph opens downwards ↶ and has a localized maximum extrema.

1st d. test



expl. My brother also had me run a first derivative test. This also showed me the shape of the graph. I chose to use -2, -.5, 1, and 5 in this test.

Questions I guess I've just been very confused by what the value of x represents when the 1st derivative is set to equal 0....

Does the graph measure distance or position? Jacob said position but I thought it was distance.

Slope

$$-\tfrac{4}{25}(-3.75)^3 + \tfrac{9}{25}(-3.75)^2 + \tfrac{4}{5}(-3.75)$$
$$-\tfrac{4}{25}(-52.734375) + \tfrac{9}{25}(14.0625) - 3$$
$$8.4375 + 5.0625 - 3 = 10.5 \checkmark \text{ OR } 10\tfrac{1}{2} \checkmark \text{ OR } \tfrac{21}{2} \checkmark$$

expl. When an x-value is entered into the 1st derivative, the resulting y-value is the slope of the individual point at that time. Because I drew my character on the graph at around the x-value -3.75, I decided to find the slope of that point. Thus the slope of that point 21/2.

Note This story assumes that the character starts at Day 1 at the x-value -4.75, reaches x-value -3.75 at Day 2, and so on until he reaches the base of the other side of the mountain after 11.75 days. I would like for this to measure distance, but my brother says this graph measures position and I admit this makes more sense based on the story.