TO INFINITY
AND BEYOND

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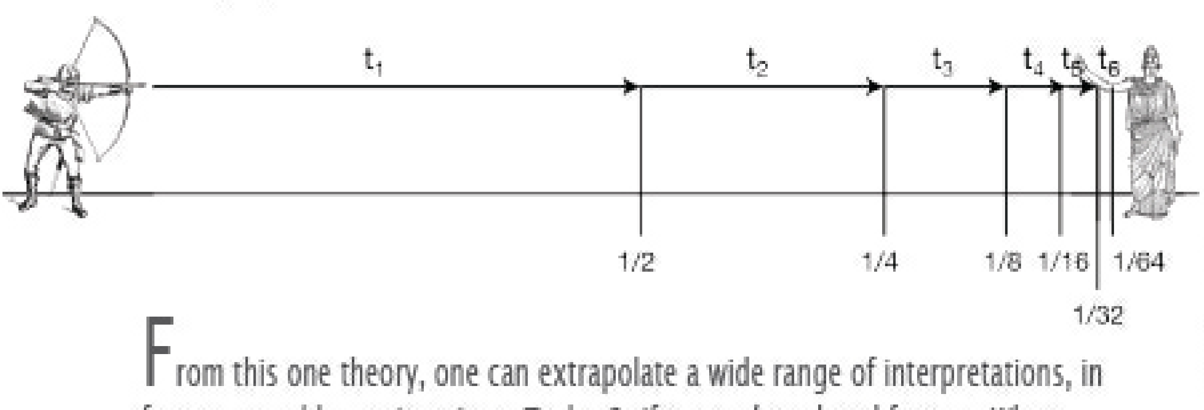
BY MARA SARGENT

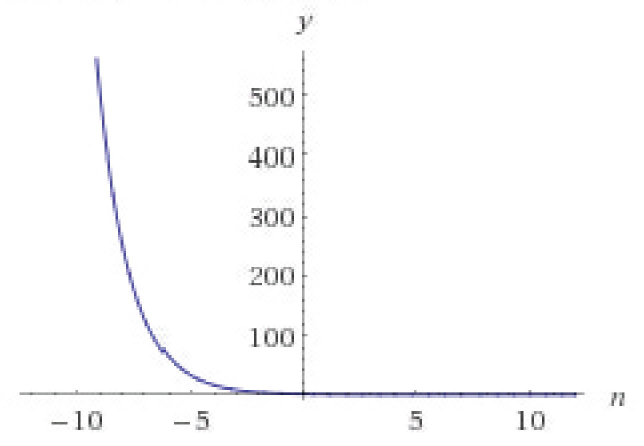
A HISTORY OF INFINITY

Where did it all begin? As it happens, the earliest time we humans have contemplated infininty was about two thousand years ago between the year of 490 BCE and 430 BCE. Greek society was characterized by a time of avant garde discoveries in philosophy, mathematics, art, and beyond. But, most importantly, the concept of **infinity** was given life during this time by a Greek philosopher named Zeno of Elea. If you're reading this, you've probably heard of Zeno's Arrow Paradox, an interesting take on the flight of the arrow. According to Zeno, an arrow actually travels an infinite distance when fired from a bow. According to his logic, once the arrow has traveled half of its required distance to reach a target, it must travel one fourth of that distance, next one eighth of that distance, and so on. Later on, Zeno's ideas were transformed into the concept we now know as infinite series, however, it was this Greek philosopher that started it all.

One day our heroic Greek philosopher Zeno procured a new mind-boggling theory that would later become the basis of the study of infinity in all of mathematics. Artistotle himself recounted the brilliant man in his compendium Physics, "If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless."



From this one theory, one can extrapolate a wide range of interpretations, in fact, one could even imagine a Taylor Swift song that played forever. What could possibly be better than that? Zeno is clearly the coolest figure in mathematics. Converting units from distance (in meters) to time (in t) along a two-dimensional axis allows one to, theoretically speaking, listen to a Taylor Swift song for the rest of their life, and beyond. If we take the song Blank Space, for example, once one has listened to one half of the song, one would still have the next one fourth of the song to hear, and then one eighth, and one sixteenth, ad infinitum. But why stop there? Zeno's work also serves as an introduction to the idea of **infinite series, divergence,** and **convergence,** two key elements in the study of infinite series. When represented in the form $\frac{1}{2^n}$ and graphed these "halvings" are clearly visible:



As you can clearly see, the distance is halving with each enumeration of n, until the series as a whole is equal to one (one half plus one fourth plus one eighth equals 4/8 + 2/8 + 1/8, which is equal to roughly 7/8, or, 0.875. As we keep adding numbers, it appears to approach one: 16/32 + 8/32 + 4/32 + 2/32 + 1/32 equals 31/32, or, 0.97. However, when represented using infinite sums, this problem evaluates to two: $\sum_{n=0}^{\infty} 2^{-n} = 2$. Infinite sums, then, are demonstrably more accurate than crude estimation. We have come far since the inception of infinity, and the field has continued to amaze both scientists and mathematicians alike in the modern world with the advent of computers and information technology.

Did you know that many popular functions can actually be represented by infinite series? More precisely, these are known as Taylor Series:

SPIRALING IN FROZEN FRACTALS ALL AROUND

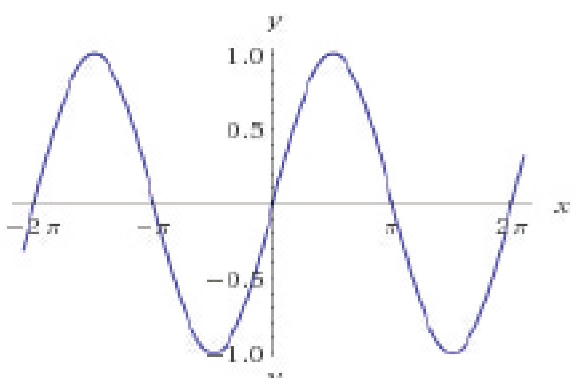
The field of infinite mathematics isn't limited to just series and sums, but is composed of a wide range of theories and ideas. Arguably the most interesting of these are **fractals**, and they exist everywhrere from the ancient texts of Greek philosophers to within popular movies like Frozen. The basic premise of a fractal is that it is a pattern that repeats itself, and the wonderful thing about them is that they can be found practically everywhere in nature. Snowflakes are a great example of this:

On a more complex level, fractals are unique, majestic creatures in that they behave in very odd ways when manipulated using mathematics. Fractals, at least most of the time, cannot be differentiated using calculus, and are actually multi-dimensional—a fractal curve is a one-dimensional line, but also contains a fractal dimension such that it both resembles a surface and line. Because of this dimensional property, fractals scale in such a way that each enumeration of a fractal is raised by its "fractal dimension," rather than the usual integer. For example, the area of a square can be represented by n^2, and the area of a sphere $4\pi r^2$. A fractal's "volume," however, can only be represented by a complex proportionality equation.

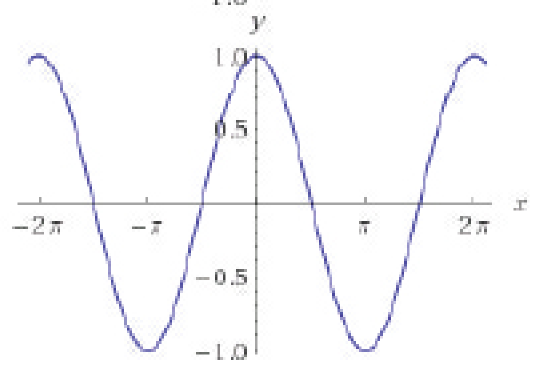
There are many unique fractals (in fact, an infinite amount), however, the most memorable ones are those that occur in nature without any outside influence. River networks, fault lines, crystals, lightning bolts, trees, the rings of Saturn, leaf formations, mountain formations, brain matter, sea shells, even our very own DNA!

Other interesting fractals include: Koch Snowflakes, Pythagoras Trees, Triflakes, 5-Circle-Inversions, Quadric Crosses, Vicsek Fractals, Pascal Triangles mod 3, Hexaflakes, and more.

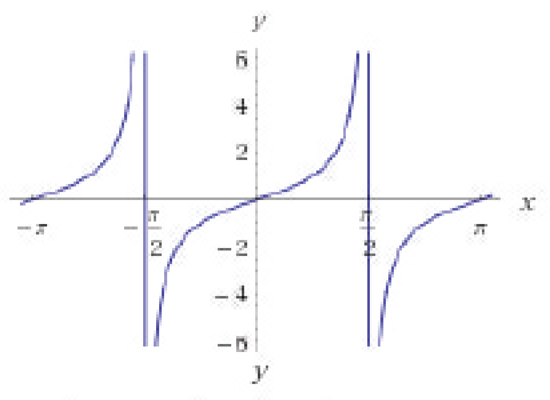
1 $$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \quad \text{for all } x$$



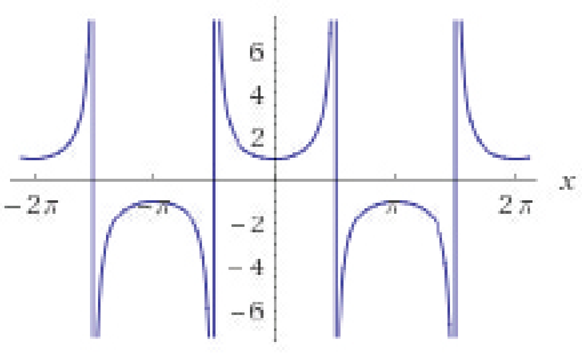
2 $$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \quad \text{for all } x$$



3 $$\tan x = \sum_{n=1}^{\infty} \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!} x^{2n-1} = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots \quad \text{for } |x| < \frac{\pi}{2}$$



4 $$\sec x = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n}}{(2n)!} x^{2n} \quad \text{for } |x| < \frac{\pi}{2}$$



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