



REAL WORLD APPLICATION

CONIC SECTIONS in life

Conic sections are everywhere--both in the natural world and the made world. Design of many different kinds benefit immensely from the science behind conic sections.

For example, the dome in the photo above, called Il Duomo di Santa Maria del Fiore, is one of Italy's largest churches, and remains the largest brick dome ever constructed. Without the basic principles of conic sections, this could not be possible.

Also, things that we take for granted, like audio speakers, owe a lot to conic sections. The parabolic structure on the speakers allows for the sound waves to focus in such a way that the sound is bettered dramatically.

Even light reflecting against walls produces a rather beautiful portrayal of conic sections.

Look around you. Conic sections are everywhere.

CONIC SECTIONS

Calcu licious
Clos ing Time

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THE BASICS

What is a CONIC SECTION?

A conic section is an intersection of a plane and a cone.

If you were to take two cones placed tip-to-tip and make a clean cut in such a way that the structure was then composed of at least two separate pieces, you would get one of four results:

- A circle
- An ellipse
- A parabola
- A hyperbola

The former two possibilities are the results of cutting straight through one of the cones so that the entirety of the base of that cone is separated from the point (and other cone).

The latter two of the above possibilities are the results of cutting the cone(s) so that the plane goes through the base(s). This also means that the base(s) are no longer circular.

In the image below, you can see that the cones with the plane intersecting at a perfectly even angle (exactly perpendicular to the central axis of the cones) create a perfectly even circle. When the plane is tilted slightly, the result is still a round shape, however, now it is warped into in a more ovalar ellipse. When the plane is brought low enough or high enough so that the plane intersects with the base, a parabola is revealed. Lastly, when a vertical cut is made which thus intersects with both cones, a hyperbola is found.



Circles and Ellipses

A circle is a type of ellipse in the same way that a square is a type of rectangle.

Any point on the circumference of the circle is an equal distance from the center. An ellipse is not necessarily this way.

The equation of a circle, while it may seem vastly different from that of an ellipse, is actually just a simplified version.

Circle equation

$$x^2 + y^2 = r^2$$

Ellipse equation

$$(x^2)/(a^2) + (y^2)/(b^2) = 1$$

While the circle equation is essentially the Pythagorean Theorem and is used to measure the radius, finding the "radius" of an ellipse is not so simple.

The terms "a" and "b" in the ellipse equation are used to represent the radius at its minimum and at its maximum, as the radius is always changing.

When the ellipse is perfectly aligned so that its center is at the origin of x and y, this making it so that lengths a and b align with either the x or y axis.

In the image above, you can see that orbits are a perfect example of circles and ellipses in the real world (or universe, in this case). In such an instance, we could use the circle and ellipse equations to discover different characteristics of an orbit such as the distance between the object in orbit and the object in the center of the orbit at all points, the speed of the orbiting object, and the distance that the orbiting object might travel in the entirety of the orbit.

Parabolas and Hyperbolas

A hyperbola is essentially two parabolas mirrored so that the round parts are facing each other. The ends of the parabolas in a hyperbola extend along asymptotes towards infinity. The equation of a parabola is:

$$y = ax^2 + bx + c$$

The equation of a hyperbola is very similar to that of an ellipse. There are two ways to graph a hyperbola:

$$(x^2)/(a^2) - (y^2)/(b^2) = 1 \quad (y^2)/(b^2) - (x^2)/(a^2) = 1$$

In the image above, you can see an example of a hyperbola in the real world. If we draw a line that goes from the upper right corner of the image to the lower left corner and another from the upper left corner to the lower right corner, they would represent the asymptotes in the image, making the hyperbola more pronounced.

